Exam Introduction to Logic (CS & MA)

Thursday 10 November, 2016, 14 - 17 h.

☞ Only write your student number at the top of the exam, not your name. Also write your student number at the top of any additional pages. In this way we will be able to grade anonymously.

☞ Use a blue or black pen (so no pencils, red pen or marker).

☞ Leave the first ten lines of the first page blank (this is where the calculation of your grade will be written).

☞ Only hand in your definite answers. You can take any drafts home.

☞ When you hand in your exam, wait until the supervisors have checked whether all information is complete. They will indicate when you can leave.

☞ By writing your student number and tutorial group on all pages, you earn a first 'free' 10 points. With the regular exercises, you can earn 90 points. With the bonus exercise, you can earn 10 extra points.
The exam grade is:
(the number of points you earned with the regular and bonus exercises + the first 'free' 10) divided by 10, with a maximum grade of 10.

☞ The final grade F for the course is computed as

$$F = 0.08 \cdot H_1 + 0.16 \cdot H_2 + 0.16 \cdot M + 0.60 \cdot E.$$

Here H_1 is the grade for homework assignment 1, H_2 is the grade for homework assignment 2, M is the midterm grade, and E is the grade for this final exam.

Good luck!

1: Translating to propositional logic (10 points) Translate the following sentences to *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key.

a. The trains rode on schedule unless it stormed or there were leaves on the tracks.

b. They will accept the outcome only if their side has won.

2: Translating to first-order logic (10 points) Translate the following sentences to *first-order logic*. Do not forget to provide the translation key. The domain of discourse is the set of all people.

a. Anand gives roses to Clara only, although Clara loves precisely those people who give roses to themselves.

b. Boris gives roses to all people who love nobody but Boris.

3: Formal proofs (20 points) Give formal proofs of the following inferences. Do not forget the justifications. You can only use the Introduction and Elimination rules and the Reiteration rule.

a.

$$\begin{array}{|l} B \to \neg C \\ \neg A \vee C \\ \hline \neg(A \wedge B) \end{array}$$

b.

$$\begin{array}{|l} \\ \hline (A \to B) \vee (A \wedge \neg B) \end{array}$$

c.

$$\begin{array}{|l} \neg\forall x(P(x) \to Q(x)) \\ \hline \exists x P(x) \end{array}$$

d.

$$\begin{array}{|l} \forall x \forall y((G(x) \wedge G(y)) \to x = y) \\ \exists z\ G(z) \\ \hline \exists w\ (G(w) \wedge \forall v(G(v) \to v = w)) \end{array}$$

4: Truth tables (10 points) Answer the following questions using truth tables. Write down the complete truth tables and motivate your answers.

a. Check with a truth table whether the conclusion is a logical consequence of the premise for the following argument. Indicate clearly which rows are spurious.

$$\begin{array}{|l} \mathsf{Large(a)} \leftrightarrow \mathsf{Medium(b)} \\ \hline \mathsf{LargerThan(a,b)} \vee \neg\mathsf{Medium(b)} \end{array}$$

b. Check with a truth table whether the following formula is a tautology.

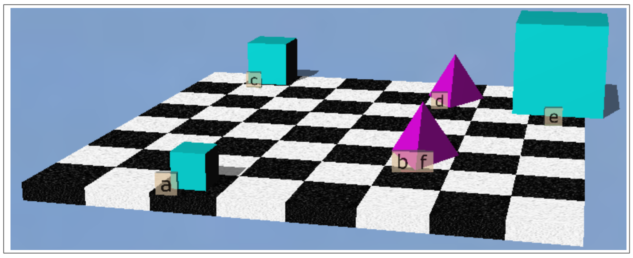
$(A \to (B \to C)) \leftrightarrow ((B \wedge A) \to C)$

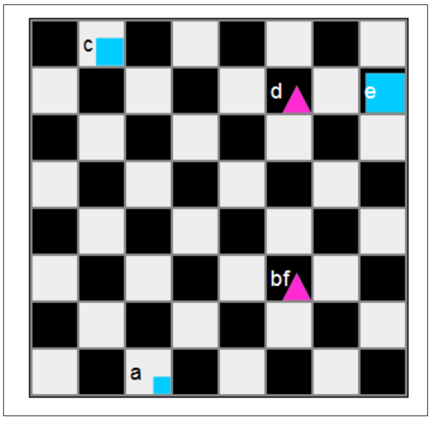
5: Normal forms propositional logic (5 points)

Provide a disjunctive normal form (DNF) of the following formula. Show all of the intermediate steps.

$$(A \vee B) \leftrightarrow (\neg B \wedge C)$$

6: Tarski's World (10 points)





In the world displayed above, a is small, e is large, and the objects named c, b, d, f are medium.

a. In the world displayed above, there are exactly two objects that are in the same row. Express this with one sentence in the language of Tarski's World. The sentence should be true in all worlds in which there are exactly two objects that are in the same row, and false in all worlds in which there are not exactly two objects that are in the same row.

b. Indicate of the sentences below, whether they are true or false in the world displayed above. You do not need to explain your answers.

(i) $(\mathsf{SameShape}(c, a) \rightarrow \mathsf{SameShape}(c, b)) \vee (\mathsf{RightOf}(e, c) \wedge \mathsf{FrontOf}(d, f))$

(ii) $\exists x\, \mathsf{Dodec}(x) \rightarrow \mathsf{Tet}(e)$

(iii) $\forall x(\mathsf{Small}(x) \rightarrow \mathsf{FrontOf}(x, b))$

(iv) $\forall x \neg \exists y((\mathsf{FrontOf}(x, y) \wedge \mathsf{Larger}(x, y)) \rightarrow \mathsf{Dodec}(x))$

c. Explain how the formula below can be made **false** by changing the position of one object in the world displayed above.

$\forall x \forall y((x \neq y \wedge \mathsf{SameColumn}(x, y) \wedge \mathsf{SameShape}(x, y) \wedge \mathsf{BackOf}(x, y)) \rightarrow \exists z\, (\mathsf{SameRow}(z, x) \wedge \mathsf{RightOf}(z, y)))$

7: Normal forms first-order logic (10 points)

a. Provide a Skolem normal form of the sentence $\neg(\forall x A(x) \rightarrow \forall x \exists y \forall z B(x, y, z))$. Show all intermediate steps.

b. Check the satisfiability of the Horn sentence. Use the Horn algorithm and indicate the order in which you assign truth values to the atomic sentences. If you prefer the conditional form, you may also use the satisfiability algorithm for conditional Horn sentences.

$$(\neg A \vee \neg B) \wedge (\neg C \vee \neg A \vee D) \wedge (\neg E \vee C) \wedge (\neg A \vee E) \wedge A$$

8: Translating function symbols (5 points) Translate the following sentences using the translation key provided. The domain of discourse is the set of all people occurring in the series "Grey's Anatomy".

mcdreamy: Dr. Shepherd
meredith: Meredith

spouse(x): x's spouse
chief(x): x's chief
mother(x): x's mother

Lovesmore(x, y, z): x loves y more than x loves z

a. Meredith's chief loves Meredith's mother more than he loves his spouse.

b. There is someone who loves both dr. Shepherd and his chief more than she loves her spouse.

9: Semantics (10 points)

Let a model $\mathfrak{M}$ with domain $\mathfrak{M}(\forall)$ = {Gödel, Russell, Frege} be given such that

- $\mathfrak{M}(a)$ = Frege
- $\mathfrak{M}(\mathsf{Consistent})$ = { Russell, Gödel }
- $\mathfrak{M}(\mathsf{Imprisoned})$ = { Russell }
- $\mathfrak{M}(\mathsf{Influenced})$ = {⟨Frege, Russell⟩, ⟨Russell, Frege⟩, ⟨Russell, Gödel⟩}

Let h be an assignment such that:

- $h(x)$ = Gödel
- $h(y)$ = Russell
- $h(z)$ = Frege.

Evaluate the following statements. Follow the truth definition step by step.

a. $\mathfrak{M} \models \mathsf{Consistent}(z) \to \mathsf{Imprisoned}(a)\ [h]$

b. $\mathfrak{M} \models \forall x\ (\mathsf{Imprisoned}(x) \leftrightarrow \neg\mathsf{Influenced}(a, x))\ [h]$

c. $\mathfrak{M} \models \exists x \forall y\ (\mathsf{Influenced}(x, y) \vee \mathsf{Consistent}(y))\ [h]$

10: Bonus question (10 points)

Give a formal proof of the following inference. Don't forget to provide justifications. You can only use the Introduction and Elimination rules and the Reiteration rule.

$$\begin{array}{|l} \neg\forall x \forall z \exists y \neg A(x, y, z) \\ \hline \exists x \exists z \forall y A(x, y, z) \wedge \forall y \exists x \exists z A(x, y, z) \end{array}$$